into the plastic flow eq. $(3.7)_1$ to yield the result

$$\dot{t}_1 - (K + \tfrac{4}{3}\mu)\dot{S}_1 = \hat{F}(\tau), \qquad (3.9)$$

where $\tau = \frac{1}{2}(t_1 - t_2)$ and $\hat{F}(\tau) = -\frac{8}{3}\mu b N V_d(\tau)$. The function $\hat{F}(\tau)$ is called the relaxation function. Equation (3.9) was derived in its present form by Duvall [64D2] and Taylor [65T2], and its thermodynamic basis has recently been discussed by Nunziato and Drumheller [78N2].

The theory of dynamic plasticity outlined accommodates nonlinear elastic response in that K and μ are allowed to depend on S_1, although several aspects of this treatment are inconsistent with modern theories of finite plastic deformation.

3.3.3. Dynamic yielding – the elastic precursor wave

The amplitude of the elastic precursor to a plastic wave provides direct evidence for the onset of plastic flow under the particular conditions of the experiment. The early work by Jones et al. [62J2], Taylor and Rice [63T2], and Ivanov et al. [63I1] has stimulated continuing investigations. These studies have been concerned with microscopic explanations of the form of elastic precursor waves and the observed decreases in their amplitude with increasing propagation distance. Both single crystal and polycrystalline materials have been studied and metallurgical condition, temperature, and impurity and initial defect concentrations have been varied.

The first detailed application of dislocation-mechanical concepts to elastic-plastic wave propagation phenomena was Taylor's [65T2] analysis of the decay of the elastic precursor wave in iron. The attenuation of this wave can be calculated without solving the equations for the entire waveform since it is identified with the variation of the stress jump along the leading characteristic for an elastic shock, a quantity that can be determined independently of the remainder of the flow field. When this shock is propagating into unstressed material, a linearized analysis gives the wavefront equation

$$\frac{\mathrm{d}}{\mathrm{d}X}(t_1^-) = \tfrac{1}{2}\hat{F}C_\ell, \quad \text{where } C_\ell = [(K + \tfrac{4}{3}\mu)/\rho^+]^{1/2}. \qquad (3.10)$$

The corresponding nonlinear result, taking account of hydrodynamic attenuation as well as stress relaxation, has been obtained by Ahrens and Duvall [66A2]. Experimental measurement of t_1^- at various propagation distances permits determination of valves of $\hat{F}$ through application of eq. (3.10). Since the relaxation function $\hat{F}$ depends on the number of dislocations participating in the yield process, the average speed with which they move, and the way in which these quantities depend on other variables, its determination provides information about the microscopic aspects of the yield process. The sensitivity of calculations to changes in the form of the velocity function is not great, but the way in which the number of mobile dislocations evolves during the passage of a wave is important and has been investigated in some detail.

Formulae relating dislocation velocity to stress that have been suggested in the literature include one derived from a reaction rate model, one based on linear damping, and one proposed by Gilman on the basis of experimental observations (see, e.g., [69R1]). Taylor adopted the Gilman form in his work, although the other relations could have been used as well since the differences among them become apparent only at very short propagation distances [67K1], or when temperature dependence or phenomena other than precursor decay rates are considered. Rohde [69R1] found that none of the dislocation-velocity formulae satisfactorily explained both the amplitude decay data of Taylor and Rice and his own observation of the temperature independence of this amplitude.

This led him to suggest twinning as a significant contributor to the deformation and subsequent work [71J2] has confirmed this hypothesis for the iron he studied.

Several investigators [71C5, 73S6, 73R1] have suggested more detailed models for calculation of dislocation velocity but the validity of such models cannot be verified without a critically-oriented experimental program. In recent work by Tyunyaev and Mineev on boron-doped single crystal silicon [76T1], it was observed that such doping did not change elastic wave attenuation. From this observation they concluded that the velocity of dislocations was controlled by non-thermally-activated mechanisms.

Holland [67H1] conducted the first dynamic yielding investigation in which the metallurgical condition of the samples was systematically varied. Observation of the effect of thermal aging of prestrained samples on the waveform propagated in Ferrovac-E iron led him to conclude that shock loading was extraordinarily effective in mobilizing pinned dislocations and/or effective in creating new ones. Subsequent work supports this conclusion but it now seems clear from work of Johnson and Rohde [71J2] and Rohde et al. [72R1] that Holland's original interpretation of his observations was in error due to neglect of twinning as an important deformation mechanism.

As pointed out by Jones and Holland [68J2], Taylor's analysis of the data of Taylor and Rice suggests that a great many dislocations participate in the yielding process, but that their average velocity is low. To investigate this point further, they measured the amplitude of elastic waves emerging from 19-mm thick samples of mild steel with grain sizes ranging from 9 to 70 μm. By comparing their data to static yielding observations and by using Taylor's dislocation parameters, they concluded that the average velocity of dislocations was low, that supersonic motion need not occur, and that yielding could be explained as a result of the conventional behavior of a large number of dislocations.

Dynamic studies in single crystals offer the advantages that slip on various systems can be activated selectively and that effects of grain boundaries and random crystallographic orientation are avoided. Investigation of single crystals of copper, NaCl, beryllium, zinc [73S6], tungsten [70J3], and LiF have been reported and the calculations required for analysis of the data prepared by Johnson et al. [70J3]. The first work in this area was undertaken by Jones and Mote [69J2], who studied copper crystals of 99.99 + % purity oriented so that shocks could be propagated in the [100], [110] and [111] directions. Unfortunately, little variation of precursor amplitude can be expected as the resolved shear stress on the primary slip system for this crystal does not vary greatly with its orientation. The ordering of the small variations that were observed was as predicted, however, and tended to confirm the hypothesis that yielding occurred when the shear stress resolved on the slip system reached a critical value. The initial dislocation density in the crystals studied was estimated to be 10^{10} m^{-2}, but a dislocation density of the order of 10^{12} to 10^{13} m^{-2} was required to explain the observed attenuation. The requirement that the density of mobile dislocations be very large if this model is to explain observed precursor attenuation rates has been found to hold in all cases considered since this early work. Its reconciliation with known initial values and credible mechanisms for increase in dislocation density has been an important objective of most subsequent research.

A rather detailed examination of the yielding of monocrystalline NaCl, a much more (plastically) anisotropic material was reported by Murri and Anderson [70M2]. They observed precursor waves of amplitude 0.027, 0.077, and 0.74 GPa emerging from 7-mm thick samples when the wave propagated in the [100], [110], and [111] direction, respectively. These data are in the order one would expect on the basis of the resolved shear stress on the primary $\{110\}\langle 110\rangle$ slip system;